Implementing the Common Core State Standards for Mathematics: The CCSS Curriculum Materials Analysis Tools

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Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents - and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimations and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying the units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x- 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x- 1)(x + 1)$, $(x - 1)(x^2 + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

McDonald's Claim

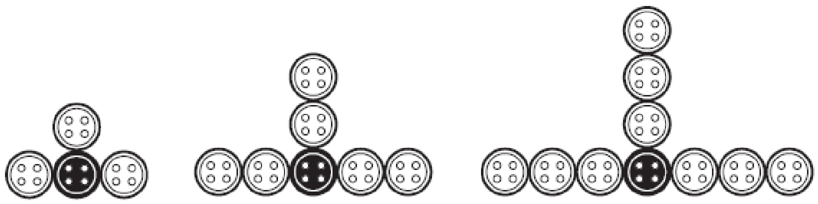
A recent Wikipedia article reports that 8% of all Americans eat at McDonald's every day. Current data indicates approximately 310 million Americans and 12,800 McDonald's restaurants in the United States.

Do you believe the Wikipedia report to be true? Create a mathematical argument to justify your position.

Buttons

This problem gives you the chance to:
- describe, extend, and make generalizations about a numeric pattern

Gita plays with her grandmother's collection of black and white buttons.
She arranges them in patterns.
Her first 3 patterns are shown below.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

1. Draw Pattern 4 next to Pattern 3.

2. How many **white** buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 ________________ Pattern 6 ________________

Explain how you figured this out.

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3. How many buttons in all does Gita need to make Pattern 11?

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Explain how you figured this out.

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____

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is **not** correct?

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How many buttons does she need to make Pattern 24? ______________

8

Name ____________________

Buttons

This problem give you the chance to:

- describe, extend, and make generalizations about a numeric pattern

Learner A

3. How many buttons in all does Gita need to make Pattern 11?

✓ 34 buttons 1

Explain how you figured this out.

I x (11 x 3) + 1 = 34 buttons

I added one for the black button 1

in the middle

Pictorial Representation

What does Learner A see staying the same? What does Learner A see as changing? Draw a picture to show how Learner A sees this pattern growing through the first 3 stages. Color coding and modeling with square tiles may come in handy.

Pattern #1 | Pattern #2 | Pattern #3

Verbal Representation

Describe in your own words how Learner A sees this pattern growing. Be sure to mention what is staying the same and what is changing.

Name ____________________

Buttons

This problem give you the chance to:

- describe, extend, and make generalizations about a numeric pattern

Learner B

3. How many buttons in all does Gita need to make Pattern 11?

4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 ____34____ ✓ 1

Explain how you figured this out.

I added 4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 0

= 34 Which is the # of buttons

Pictorial Representation

What does Learner B see staying the same? What does Learner B see as changing? Draw a picture to show how Learner B sees this pattern growing through the first 3 stages. Color coding and modeling with square tiles may come in handy.

Pattern #1 Pattern #2 Pattern #3

Verbal Representation

Describe in your own words how Learner B sees this pattern growing. Be sure to mention what is staying the same and what is changing.

Levels of Demand

Lower-level demands

Memorization:

a. Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
b. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
c. Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
d. Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Procedures without connections:

a. Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
b. Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
c. Have no connection to the concepts or meaning that underlie the procedures being used.
d. Are focused on producing correct answers vs developing mathematical understanding.
e. Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands

Procedures with connections:

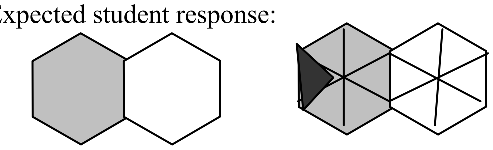
a. Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
b. Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
c. Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols and problem situations. Making connections among multiple representations helps develop meaning.
d. Requires some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Doing mathematics:

a. Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
b. Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
c. Demand self-monitoring or self-regulation of one's own cognitive processes.
d. Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
e. Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
f. Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

From: Smith, M.S., & Stein, M.K. (1998). Mathematical tasks as a framework for reflection: From research to practice. Mathematics Teaching in the Middle School,, 3(4), pp. 268-275.

Examples of Higher and Lower Cognitive Demand Tasks

Lower-Level Demands	Higher-Level Demands
Memorization What is the rule for multiplying fractions? Expected student response: You multiply the numerator times the numerator and the denominator times the denominator. or You multiply the two top numbers and then the two bottom numbers. ***Procedures without Connections*** Multiply: $\frac{2}{3} x \frac{3}{4}$ $\frac{5}{6} x \frac{7}{8}$ $\frac{4}{9} x \frac{3}{5}$ Expected student response: $\frac{2}{3} x \frac{3}{4} = \frac{2x3}{3x4} = \frac{6}{12}$ $\frac{5}{6} x \frac{7}{8} = \frac{5x7}{6x8} = \frac{35}{48}$ $\frac{4}{9} x \frac{3}{5} = \frac{4x3}{9x5} = \frac{12}{45}$ Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase? Expected student response: The formula for area is l x w. 15 x 10 = 150. She will need 150 square feet of carpet.	***Procedures with Connections*** Find $\frac{1}{6}$ of $\frac{1}{2}$. Use pattern blocks. Draw your answer and explain your solution. Expected student response:  First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So 1/6 of 1/2 is 1/12. ***Doing Mathematics*** Create a real-world situation for the following problem; $\frac{2}{3} x \frac{3}{4}$. Solve the problem you have created without using the rule, and explain your solution. One possible student response: For lunch Mom gave me three-fourths of a pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat? I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be only two of the shaded sections.